(a) Find the limiting velocity of a solid sphere of radius $a$ and density $\rho$ falling freely in a medium of density $\rho'$ and coefficient of viscosity $\mu$.
(b) In 1910 R. A. Millikan studied the motion of tiny droplets of oil falling in an electric field. A field of strength $E$ exerts a force $Ee$ on a droplet with charge $e$. Assume that $E$ has been adjusted, so the droplet is held stationary ($v = 0$), and that $w$ and $B$ are as given above. Find an expression for $e$. Millikan repeated this experiment many times, and from the data that he gathered he was able to deduce the charge on an electron.

30. A mass of 0.40 kg is dropped from rest in a medium offering a resistance of $0.2|v|$, where $v$ is measured in meters per second.

(a) If the mass is dropped from a height of 25 m, find its velocity when it hits the ground.
(b) If the mass is to attain a velocity of no more than 8 m/s, find the maximum height from which it can be dropped.
(c) Suppose that the resistive force is $k|v|$, where $v$ is measured in meters per second and $k$ is a constant. If the mass is dropped from a height of 25 m and must hit the ground with a velocity of no more than 8 m/s, determine the coefficient of resistance $k$ that is required.

31. Suppose that a rocket is launched straight up from the surface of the earth with initial velocity $v_0 = \sqrt{2gR}$, where $R$ is the radius of the earth. Neglect air resistance.

(a) Find an expression for the velocity $v$ in terms of the distance $x$ from the surface of the earth.
(b) Find the time required for the rocket to go 240,000 mi (the approximate distance from the earth to the moon). Assume that $R = 4{,}000$ mi.

32. Let $v(t)$ and $w(t)$, respectively, be the horizontal and vertical components of the velocity of a batted (or thrown) baseball. In the absence of air resistance, $v$ and $w$ satisfy the equations

$$dv/dt = 0, \qquad dw/dt = -g.$$

(a) Show that

$$v = u\cos A, \qquad w = -gt + u\sin A,$$

where $u$ is the initial speed of the ball and $A$ is its initial angle of elevation.
(b) Let $x(t)$ and $y(t)$, respectively, be the horizontal and vertical coordinates of the ball at time $t$. If $x(0) = 0$ and $y(0) = h$, find $x(t)$ and $y(t)$ at any time $t$.
(c) Let $g = 32$ ft/s$^2$, $u = 125$ ft/s, and $h = 3$ ft. Plot the trajectory of the ball for several values of the angle $A$; that is, plot $x(t)$ and $y(t)$ parametrically.
(d) Suppose the outfield wall is at a distance $L$ and has height $H$. Find a relation between $u$ and $A$ that must be satisfied if the ball is to clear the wall.
(e) Suppose that $L = 350$ ft and $H = 10$ ft. Using the relation in part (d), find (or estimate from a plot) the range of values of $A$ that corresponds to an initial velocity of $u = 110$ ft/s.
(f) For $L = 350$ ft and $H = 10$ ft, find the minimum initial velocity $u$ and the corresponding optimal angle $A$ for which the ball will clear the wall.

33. A more realistic model (than that in Problem 32) of a baseball in flight includes the effect of air resistance. In this case, the equations of motion are

$$dv/dt = -rv, \qquad dw/dt = -g - rw,$$

where $r$ is the coefficient of resistance.

(a) Determine $v(t)$ and $w(t)$ in terms of initial speed $u$ and initial angle of elevation $A$.
(b) Find $x(t)$ and $y(t)$ if $x(0) = 0$ and $y(0) = h$.
(c) Plot the trajectory of the ball for $r = \frac{1}{5}$, $u = 125$, $h = 3$, and for several values of $A$. How do the trajectories differ from those in Problem 32 with $r = 0$?
(d) Assuming that $r = \frac{1}{5}$ and $h = 3$, find the minimum initial velocity $u$ and the optimal angle $A$ for which the ball will clear a wall that is 350 ft distant and 10 ft high. Compare this result with that in Problem 32(f).

34. **Brachistochrone Problem.** One of the famous problems in the history of mathematics is the brachistochrone[3] problem: to find the curve along which a particle will slide without friction in the minimum time from one given point $P$ to another $Q$, the second point being lower than the first but not directly beneath it (see Figure 2.3.6). This problem was posed by Johann Bernoulli in 1696 as a challenge problem to the mathematicians of his day. Correct solutions were found by Johann Bernoulli and his brother Jakob Bernoulli and by Isaac Newton, Gottfried Leibniz, and the Marquis de L'Hôpital. The brachistochrone problem is important in the development of mathematics as one of the forerunners of the calculus of variations.

In solving this problem, it is convenient to take the origin as the upper point $P$ and to orient the axes as shown in Figure 2.3.6. The lower point $Q$ has coordinates $(x_0, y_0)$. It is then

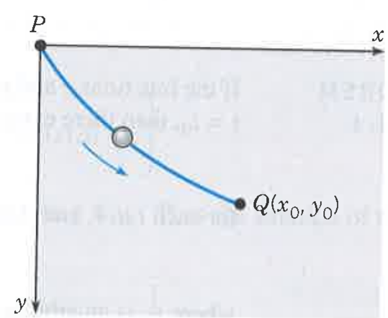


**FIGURE 2.3.6** The brachistochrone.

[3]The word "brachistochrone" comes from the Greek words *brachistos*, meaning shortest, and *chronos*, meaning time.