Just as in Example 2, we substitute $\phi(t)$ into $y'' + y = 0$ to find that

$$\underbrace{-2\cos t + 3\sin t}_{\phi''(t)} + \underbrace{2\cos t - 3\sin t}_{\phi(t)} = 0$$

for all $t$. Therefore $\phi(t)$ is a solution of $y'' + y = 0$ on $-\infty < t < \infty$. Next we must check to see if the initial conditions specified in the initial value problem (29) are satisfied. Since

$$\phi(0) = 2\cos 0 - 3\sin 0 = 2$$

and

$$\phi'(0) = -2\sin 0 - 3\cos 0 = -3,$$

we conclude that $\phi(t)$ is a solution of the initial value problem (29).

## PROBLEMS

In each of Problems 1 through 6, determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$
2. $(1 + y^2)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$
3. $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$
4. $\dfrac{dy}{dt} + ty^2 = 0$
5. $\dfrac{d^2y}{dt^2} + \sin(t + y) = \sin t$
6. $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (\cos^2 t)y = t^3$

Show that Eq. (10) can be matched to each equation in Problems 7 through 12 by a suitable choice of $n$, coefficients $a_0, a_1, \ldots, a_n$, and function $g$. In each case, state whether the equation is homogeneous or nonhomogeneous.

7. $\dfrac{dQ}{dt} = -\left(\dfrac{1}{1+t}\right)Q + 2\sin t$
8. $\dfrac{d^2y}{dt^2} = ty$
9. $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = \ln x, \quad x > 0$
10. $\dfrac{d}{dx}\left[(1 - x^2)\dfrac{d}{dx}P_n\right] + n(n+1)P_n = 0, \quad n$ constant
11. $\dfrac{d^4y}{dt^4} + (\cos t)\dfrac{d^2y}{dt^2} + y = e^{-t}\sin t$
12. $\dfrac{d}{dx}\left[p(x)\dfrac{dy}{dx}\right] - q(x)y + \lambda r(x)y = 0, \quad \lambda$ constant

In each of Problems 13 through 20, verify that each given function is a solution of the differential equation.

13. $y'' - y = 0; \quad y_1(t) = e^t, \quad y_2(t) = \cosh t$
14. $y'' + 2y' - 3y = 0; \quad y_1(t) = e^{-3t}, \quad y_2(t) = e^t$
15. $ty' - y = t^2; \quad y = 3t + t^2$
16. $y'''' + 4y''' + 3y = t; \quad y_1(t) = t/3,$ $y_2(t) = e^{-t} + t/3$
17. $2t^2y'' + 3ty' - y = 0, \quad t > 0; \quad y_1(t) = t^{1/2},$ $y_2(t) = t^{-1}$
18. $t^2y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2},$ $y_2(t) = t^{-2}\ln t$
19. $y'' + y = \sec t, \quad 0 < t < \pi/2;$ $y = (\cos t)\ln\cos t + t\sin t$
20. $y' - 2ty = 1; \quad y = e^{t^2}\int_0^t e^{-s^2}\,ds + e^{t^2}$

In each of Problems 21 through 24, determine the values of $r$ for which the given differential equation has solutions of the form $y = e^{rt}$.

21. $y' + 2y = 0$
22. $y'' - y = 0$
23. $y'' + y' - 6y = 0$
24. $y''' - 3y'' + 2y' = 0$

In each of Problems 25 and 26, determine the values of $r$ for which the given differential equation has solutions of the form $y = t^r$ for $t > 0$.

25. $t^2y'' + 4ty' + 2y = 0$
26. $t^2y'' - 4ty' + 4y = 0$

In Problems 27 through 31, verify that $y(t)$ satisfies the given differential equation. Then determine a value of the constant $C$ so that $y(t)$ satisfies the given initial condition.

27. $y' + 2y = 0; \quad y(t) = Ce^{-2t}, \quad y(0) = 1$
28. $y' + (\sin t)y = 0; \quad y(t) = Ce^{\cos t}, \quad y(\pi) = 1$

29. $y' + (2/t)y = (\cos t)/t^2;\quad y(t) = (\sin t)/t^2 + C/t^2$, $y(1) = \frac{1}{2}$

30. $ty' + (t+1)y = t;\quad y(t) = (1 - 1/t) + Ce^{-t}/t$, $y(\ln 2) = 1$

31. $2y' + ty = 2;\quad y = e^{-t^2/4}\int_0^t e^{s^2/4}\,ds + Ce^{-t^2/4}$, $y(0) = 1$

32. Verify that the function $\phi(t) = c_1 e^{-t} + c_2 e^{-2t}$ is a solution of the linear equation

$$y'' + 3y' + 2y = 0$$

for any choice of the constants $c_1$ and $c_2$. Determine $c_1$ and $c_2$ so that each of the following initial conditions is satisfied:

**(a)** $y(0) = -1,\quad y'(0) = 4$
**(b)** $y(0) = 2,\quad y'(0) = 0$

33. Verify that the function $\phi(t) = c_1 e^t + c_2 te^t$ is a solution of the linear equation

$$y'' - 2y' + y = 0$$

for any choice of the constants $c_1$ and $c_2$. Determine $c_1$ and $c_2$ so that each of the following initial conditions is satisfied:

**(a)** $y(0) = 3,\quad y'(0) = 1$
**(b)** $y(0) = 1,\quad y'(0) = -4$

34. Verify that the function $\phi(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t$ is a solution of the linear equation

$$y'' + 2y' + 5y = 0$$

for any choice of the constants $c_1$ and $c_2$. Determine $c_1$ and $c_2$ so that each of the following initial conditions is satisfied:

**(a)** $y(0) = 1,\quad y'(0) = 1$
**(b)** $y(0) = 2,\quad y'(0) = 5$