11. $dy/dt = ay - b\sqrt{y}, \quad a > 0, \quad b > 0, \quad y_0 \geq 0$

12. $dy/dt = y^2(4 - y^2), \quad -\infty < y_0 < \infty$

13. $dy/dt = y^2(1 - y)^2, \quad -\infty < y_0 < \infty$

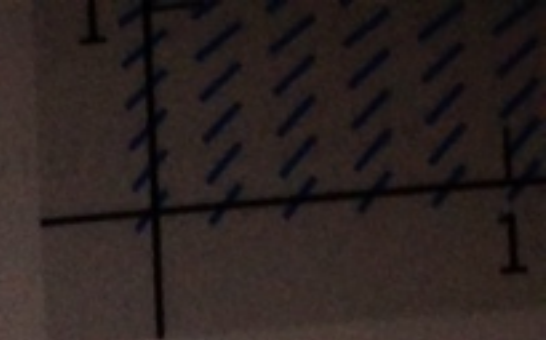


FIGURE 1.2.9

Direction Fields. In each of Problems 14 through 19 draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.

14. $y' = 3 - 2y$
15. $y' = 2y - 3$
16. $y' = 3 + 2y$
17. $y' = -1 - 2y$
18. $y' = 1 + 2y$
19. $y' = y + 2$

In each of Problems 20 through 23 draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that in these problems the equations are not of the form $y' = ay + b$, and the behavior of their solutions is somewhat more complicated than the solutions shown in Figures 1.2.6 and 1.2.7.

20. $y' = y(4 - y)$
21. $y' = -y(5 - y)$
22. $y' = y^2$
23. $y' = y(y - 2)^2$

Consider the following list of differential equations, some of which produced the direction fields shown in Figures 1.2.9 through 1.2.14. In each of Problems 24 through 29 identify the differential equation that corresponds to the given direction field.

(a) $y' = 2y - 1$
(b) $y' = 2 + y$
(c) $y' = y - 2$
(d) $y' = y(y + 3)$
(e) $y' = y(y - 3)$

25. The direction fie

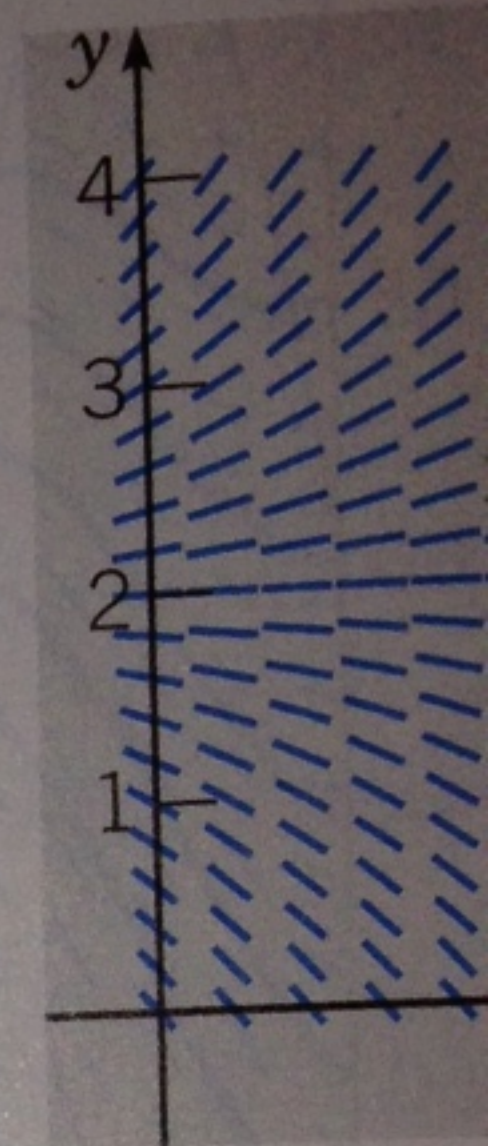


FIGURE 1.2.10

26. The direction fi

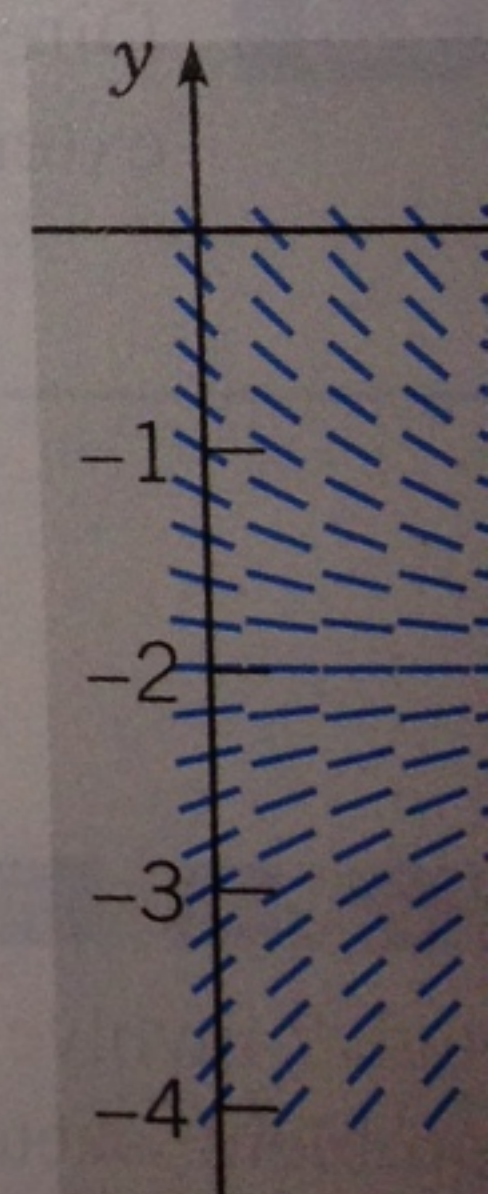


FIGURE 1.2.11

$-\infty < y_0 < \infty$

$b > 0, \quad y_0 \geq 0$

$b > 0, \quad -\infty < y_0 < \infty$

equations of the form
tch the graph of $f(y)$ ver-
rium) points, and classify
unstable, or semistable.
veral graphs of solutions

$-\infty < y_0 < \infty$

$y_0 < \infty$

$y_0 < \infty$

$b > 0, \quad y_0 \geq 0$

$y_0 < \infty$

$y_0 < \infty$

ms 14 through 19 draw
ntial equation. Based on
avior of y as $t \to \infty$. If
value of y at $t = 0$, de-

w a direction field for
on the direction field,
. If this behavior de-
, describe this depen-
quations are not of the
eir solutions is some-
ons shown in Figures

al equations, some of
own in Figures 1.2.9
through 29 identify
ds to the given direc-

(f) $y' = 1 + 2y$
(g) $y' = -2 - y$
(h) $y' = y(3 - y)$
(i) $y' = 1 - 2y$
(j) $y' = 2 - y$

24. The direction field of Figure 1.2.9.

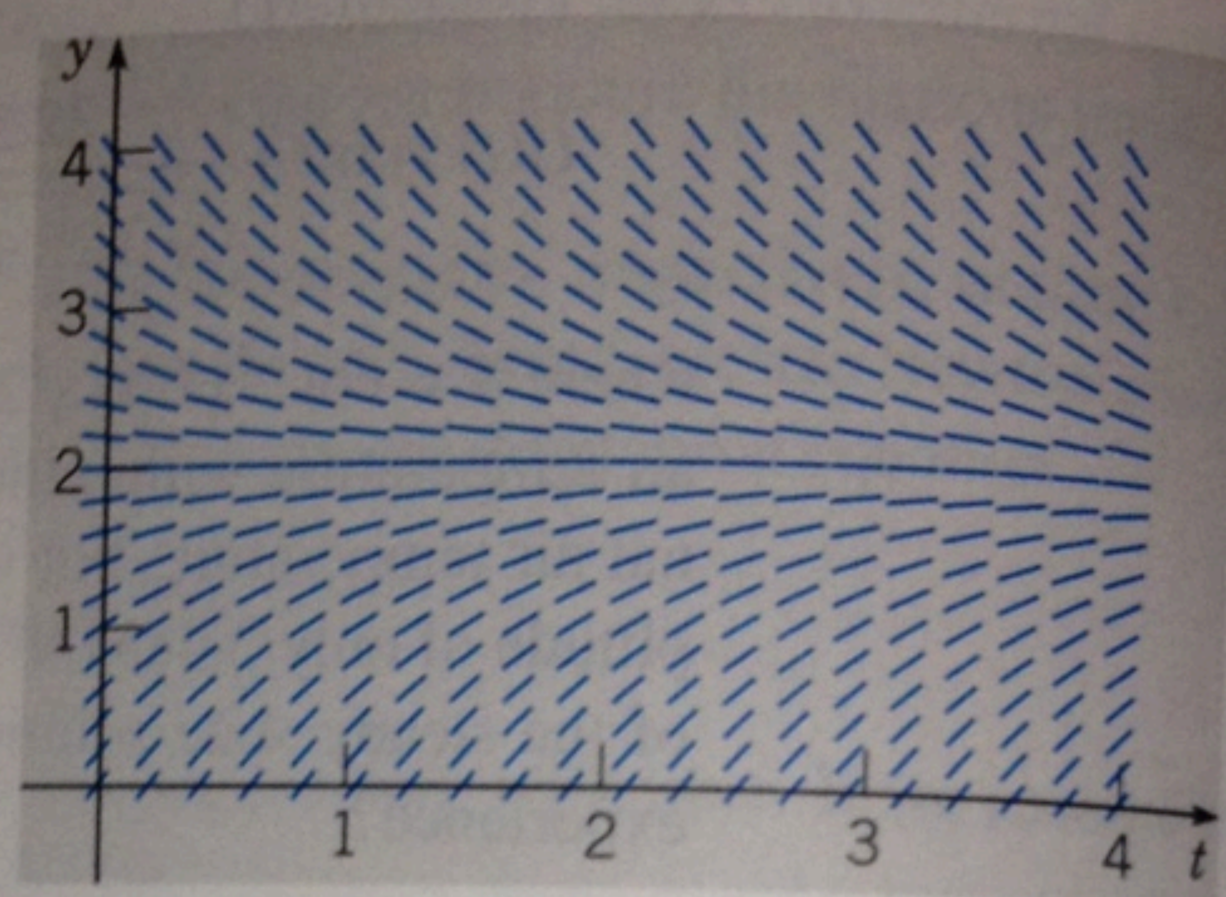


FIGURE 1.2.9 Direction field for Problem 24.

25. The direction field of Figure 1.2.10.

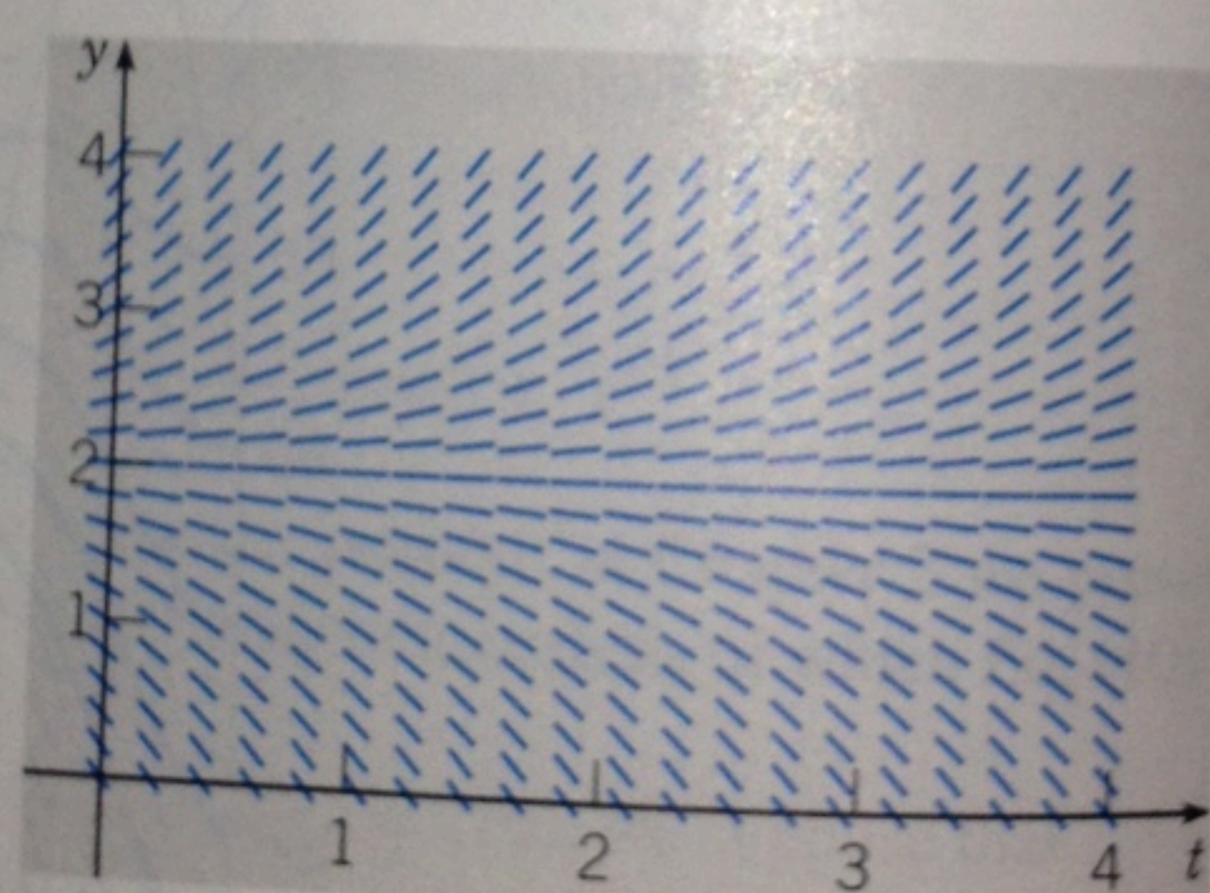
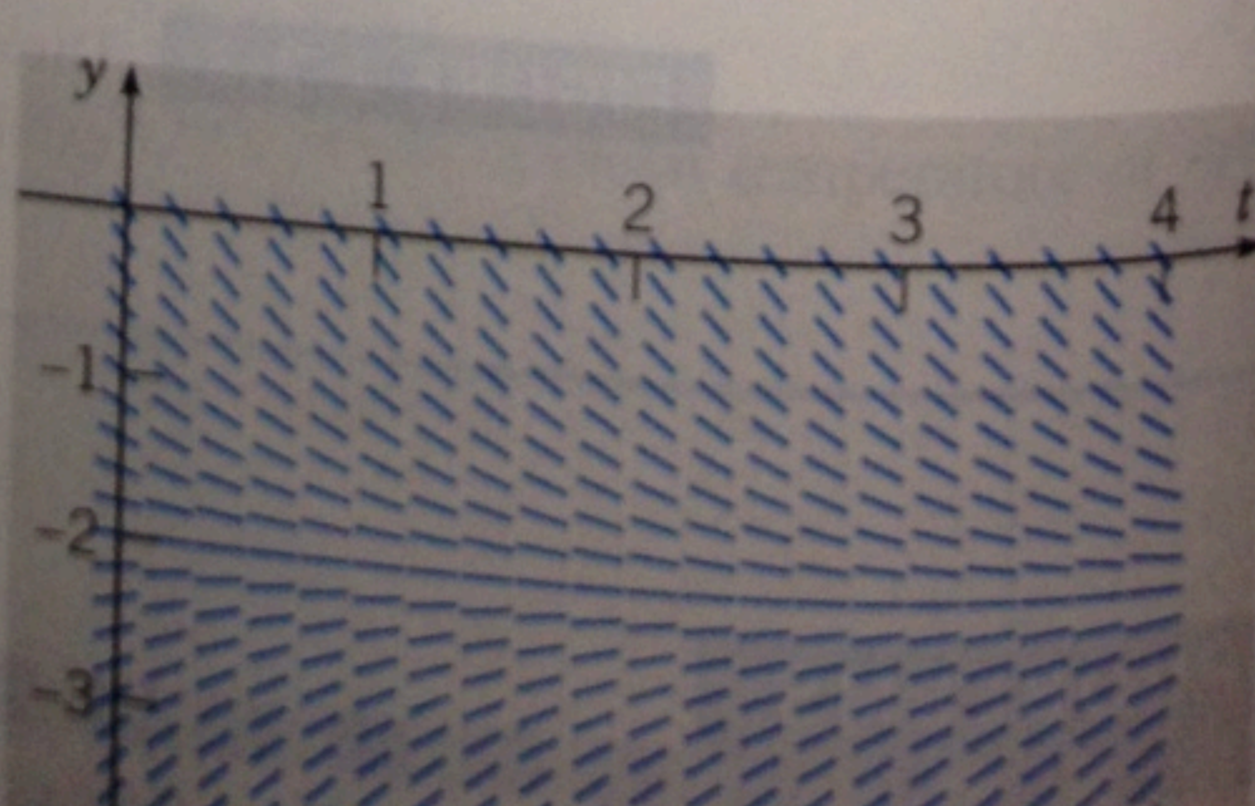


FIGURE 1.2.10 Direction field for Problem 25.

26. The direction field of Figure 1.2.11.



27. The direction field

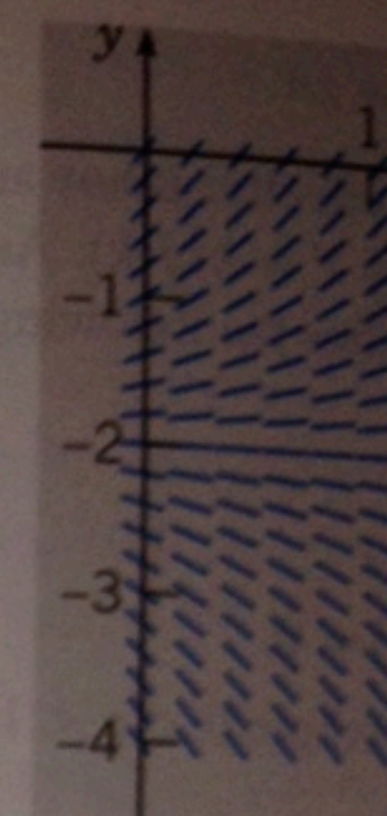


FIGURE 1.2.12

28. The direction field

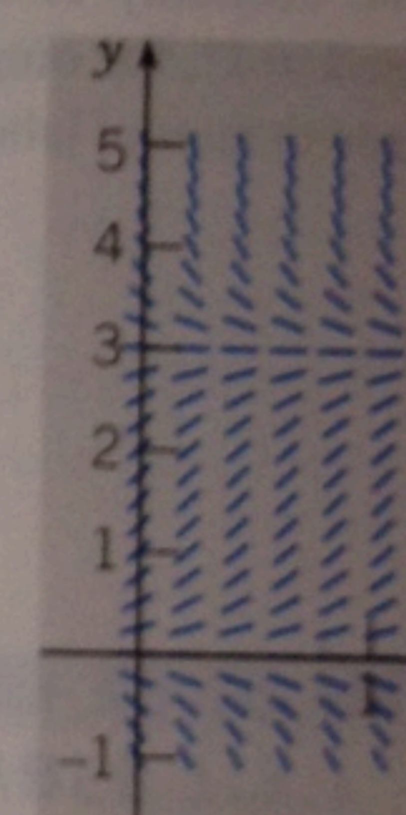


FIGURE 1.2.13 D

29. The direction field

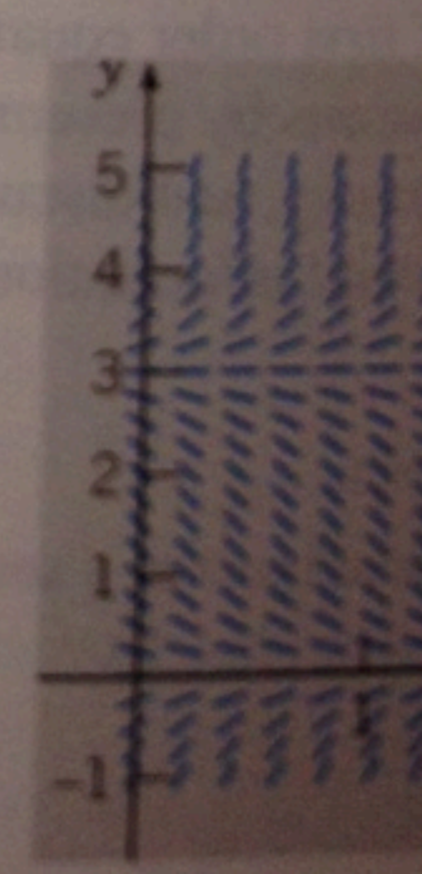


FIGURE 1.2.14 D

30. Verify that the functi
(10).

27. The direction field of Figure 1.2.12.

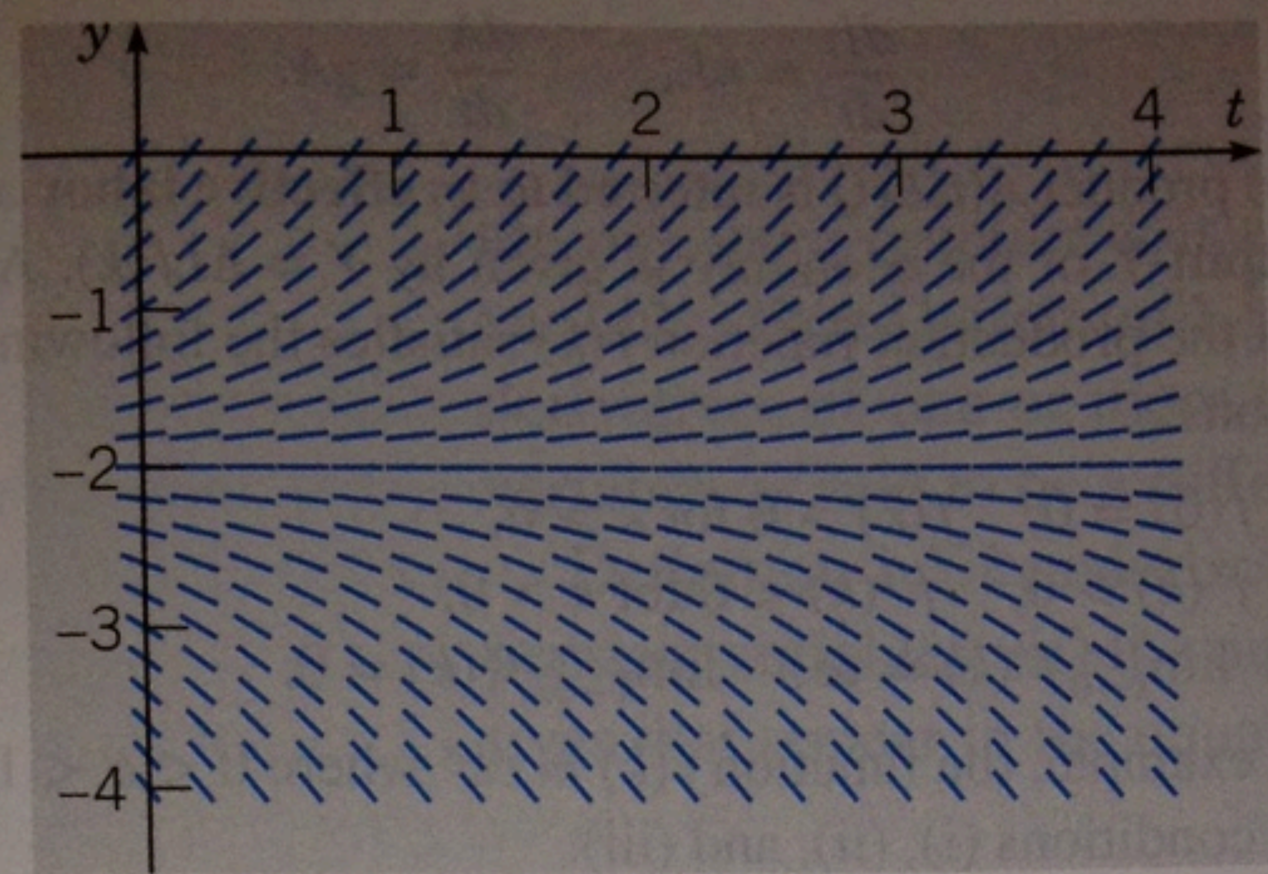


FIGURE 1.2.12 Direction field for Problem 27.

28. The direction field of Figure 1.2.13.

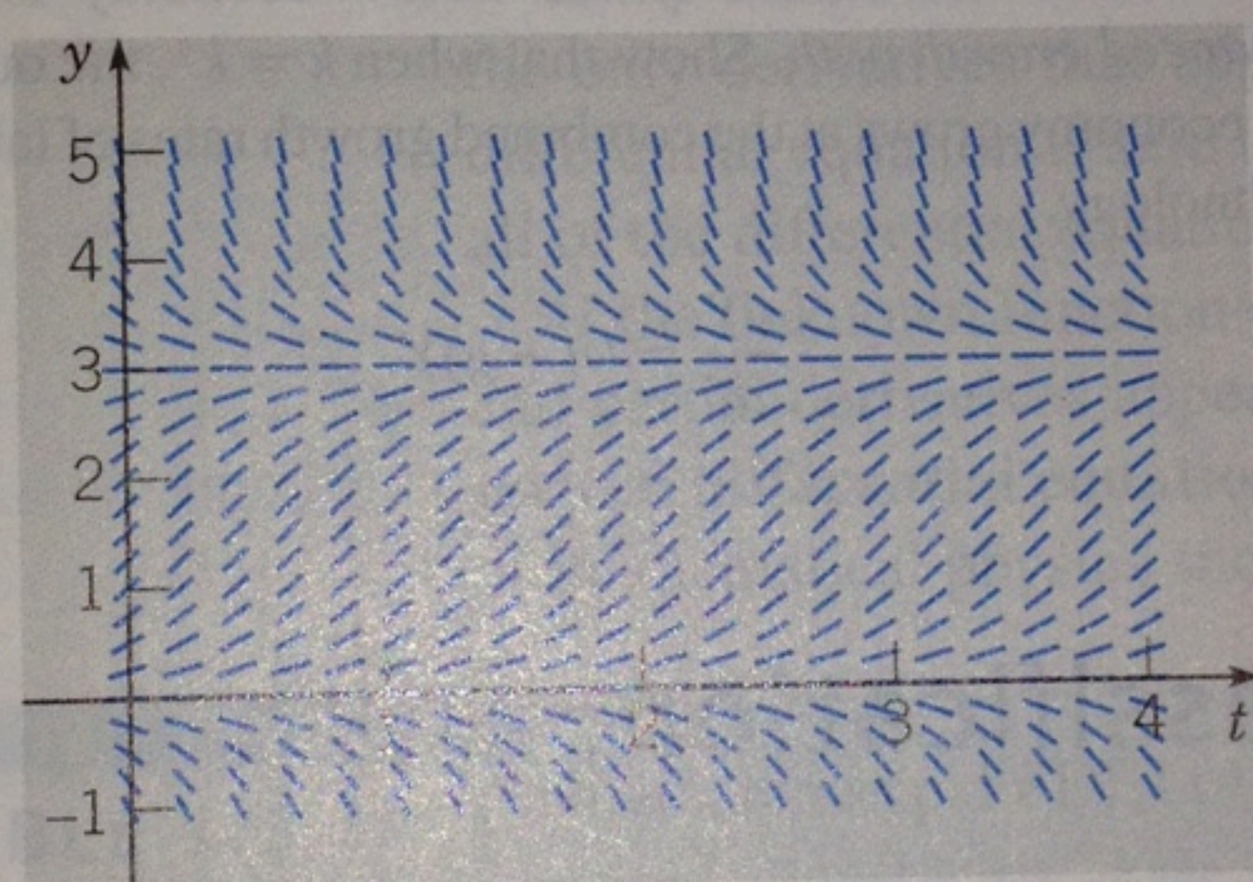


FIGURE 1.2.13 Direction field for Problem 28.

29. The direction field of Figure 1.2.14.

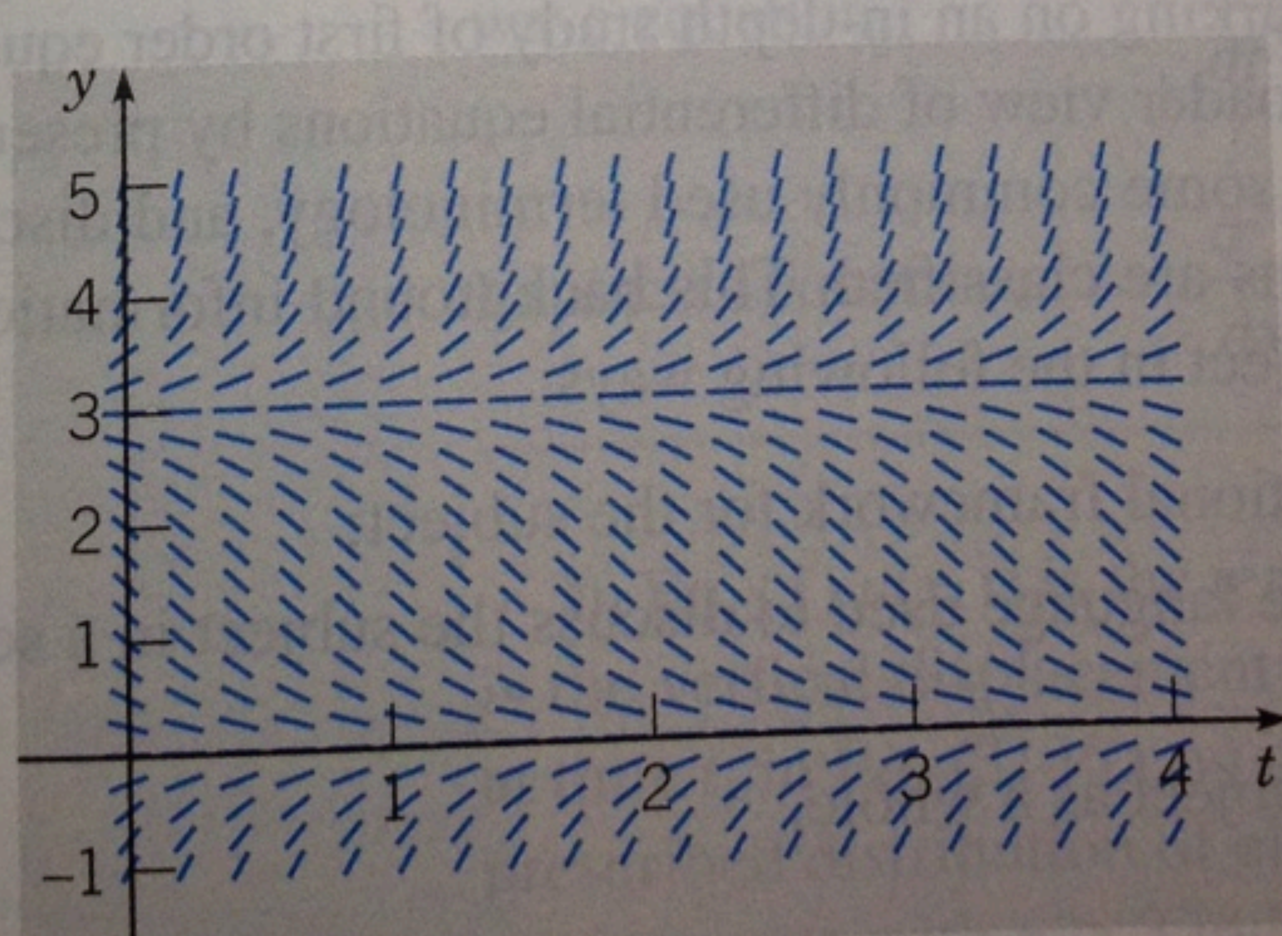


FIGURE 1.2.14 Direction field for Problem 29.

30. Verify that the function in Eq. (11) is a solution of Eq.

Applications.

32. If in the exponential mod $dy/dt = ry$, the constant growth r rate $r(1 - y/K)$ that decreases lir ulation increases, we obtain the l growth,

$$\frac{dy}{dt} = ry\Big(1$$

in which K is referred to as the c ulation. Sketch the graph of $f(y)$ determine whether each is asym

33. An equation that is freque ulation growth of cancer cells equation

$$\frac{dy}{dt} = ry$$

where r and K are positive cor

(a) Sketch the graph of $f(y)$ ve and determine whether each i stable.

(b) For each y in $0 < y \leq K$, s Gompertz equation, is never l logistic equation, Eq. (i) in Pr

34. In addition to the Gompe another equation used to moc mors is the Bertalanffy equat

$$\frac{dV}{dt} = a$$

where a and b are positive that the tumor grows at a ra while the loss of tumor mass to the volume of the tumor. sus V, find the critical points asymptotically stable or uns

35. A chemical of fixed co tinuously stirred tank reacto r_i and flows out at the sam chemical undergoes a simpl at a rate proportional to the

$$\frac{dc}{dt} = \frac{r_i}{V}$$

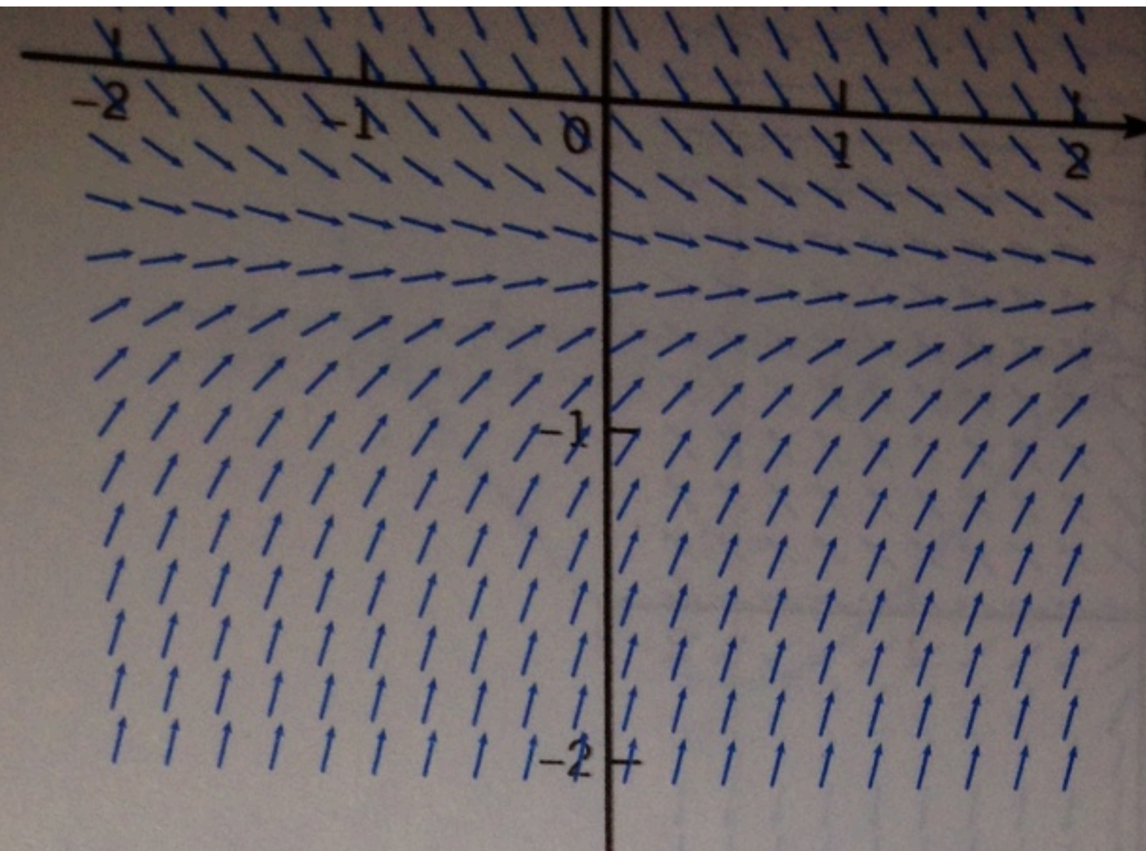
where V is the volume of th action.

(a) Use the dimensionless

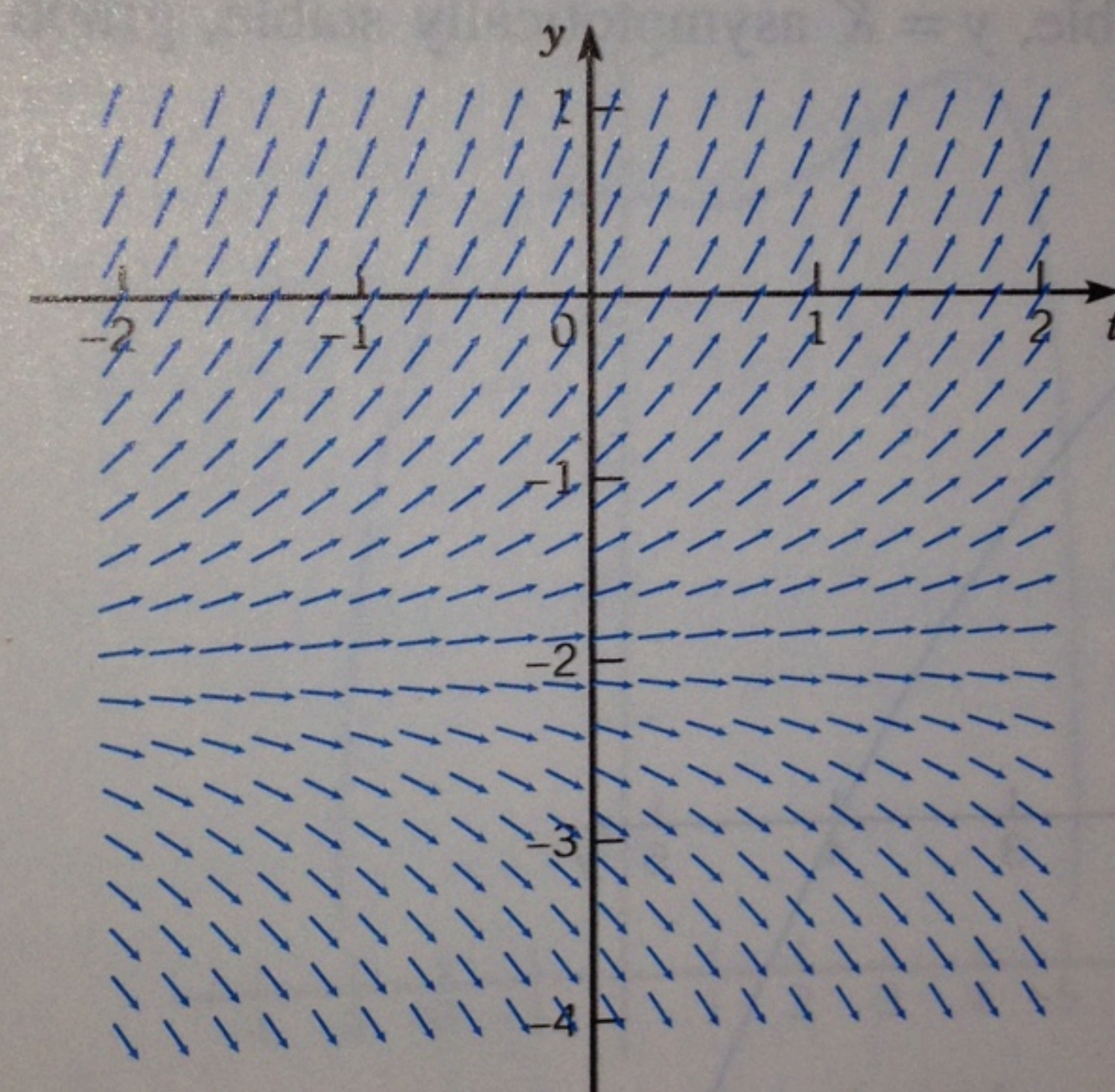
$$C = \frac{c}{c_i},$$

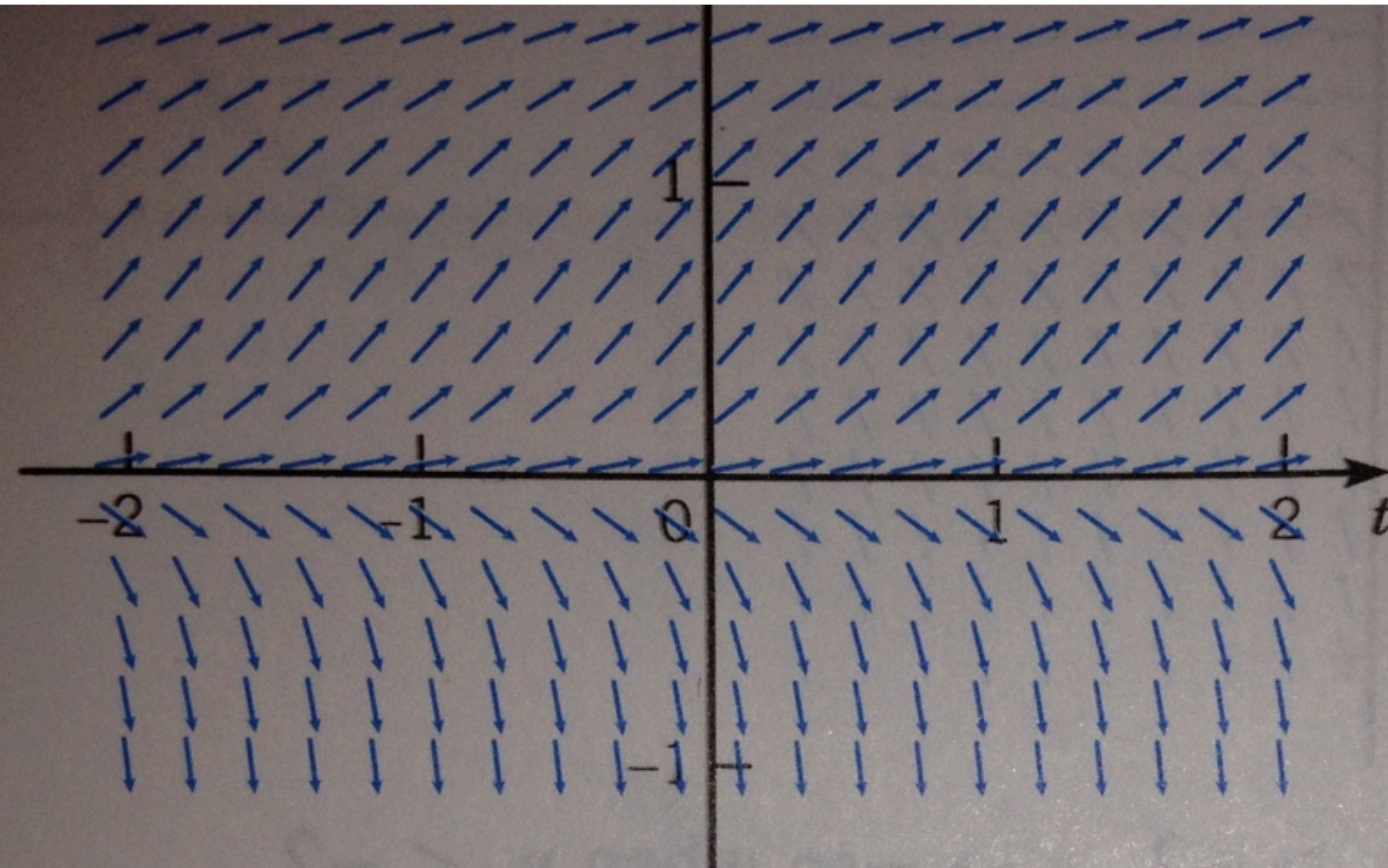
to express Eq. (i) in dimens

$$\frac{dC}{}=$$



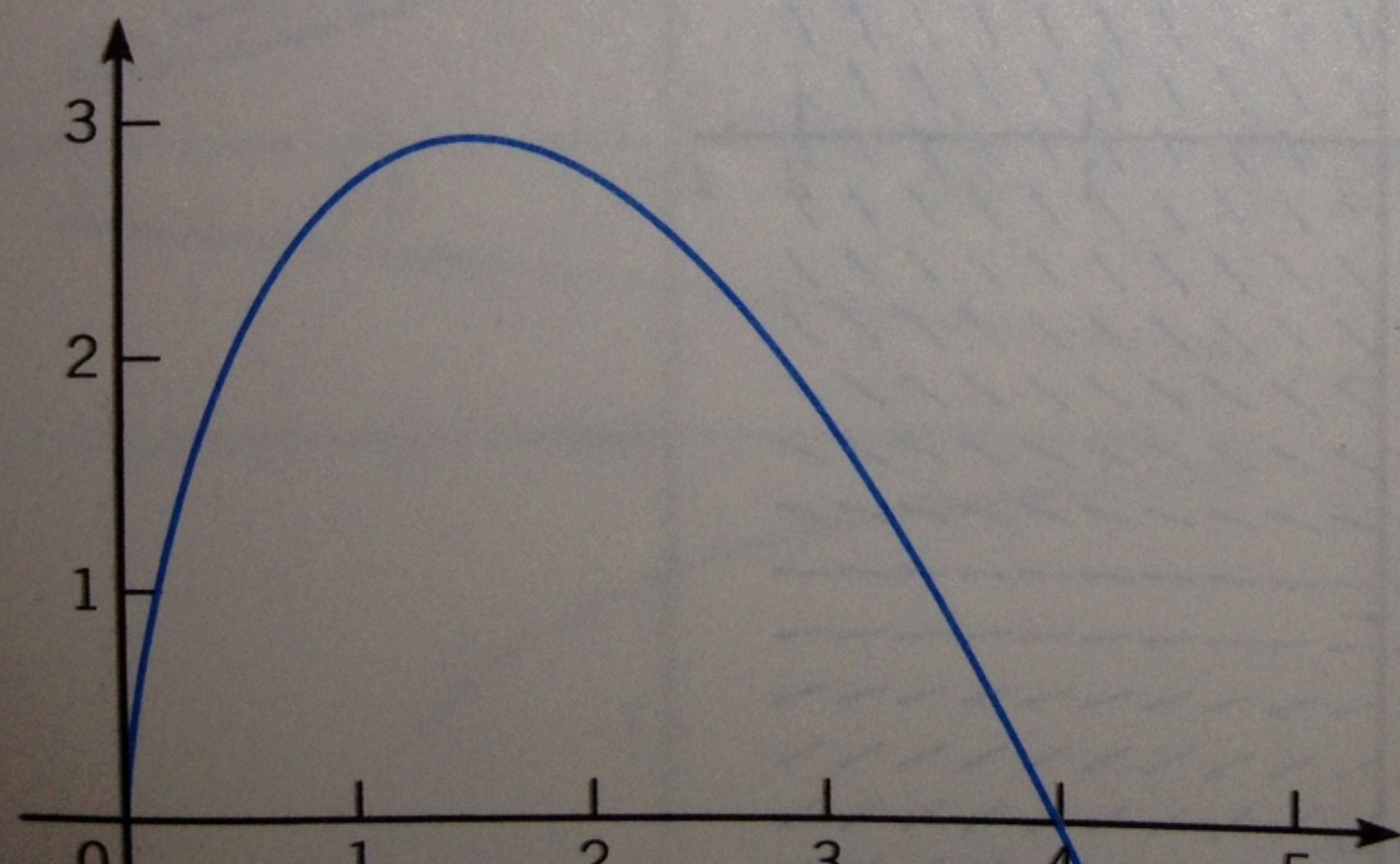
19. $y \to \infty$ when $y_0 > -2$, $y \to -\infty$ when $y_0 < -2$, $y \to -2$ when $y_0 = -2$





25. (c) **27.** (b) **29.** (e)

33. (a) $y = 0$ unstable, $y = K$ asymptotically stable, graph depicts $K = 4$



21.

23.

25.

27.

29.

31.

33.

CH

EQ

Sec

1. 5

3. y

5. 8

±(2n

7. y^2

9. 3y