A Crash Course in Geometric Structures on Surfaces

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Sage Days
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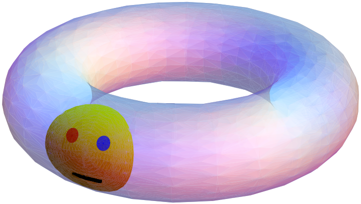
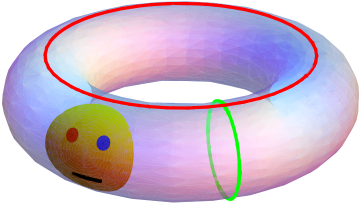
Torus Structures

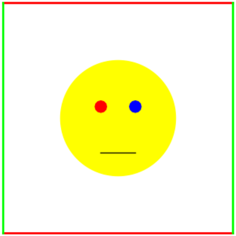
Mr. Smiley lives on a torus and we wish to study the geometry of Mr. Smiley's world



Cut the surface along 2 simple closed geodesics which intersect once (geometric intersection number 1)

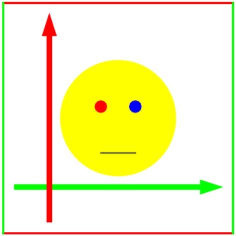


Unfolding we obtain a square:



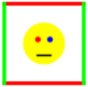
Note that all the boundaries of the square are Euclidean geodesic segments (duh!)

The original torus can be obtained by gluing opposite sides by translations



These are Euclidean Isometries

Build the Universal Cover for the Torus



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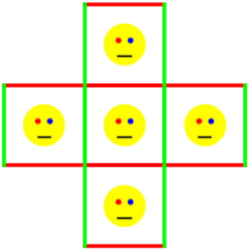

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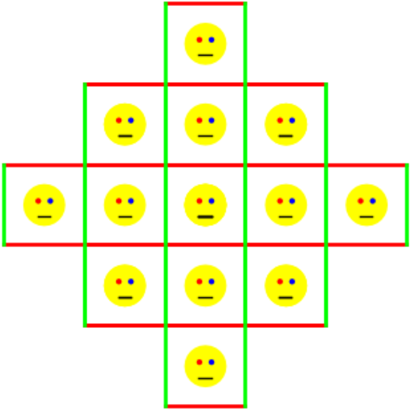

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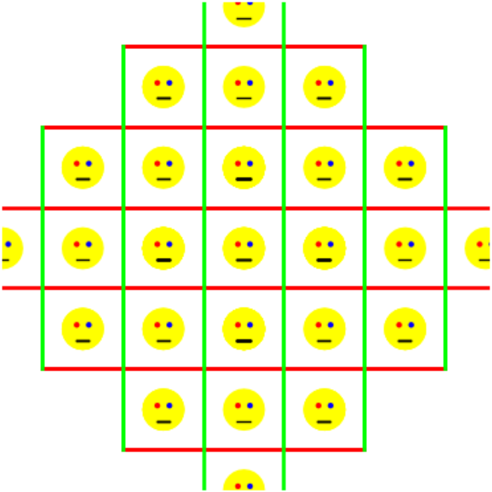

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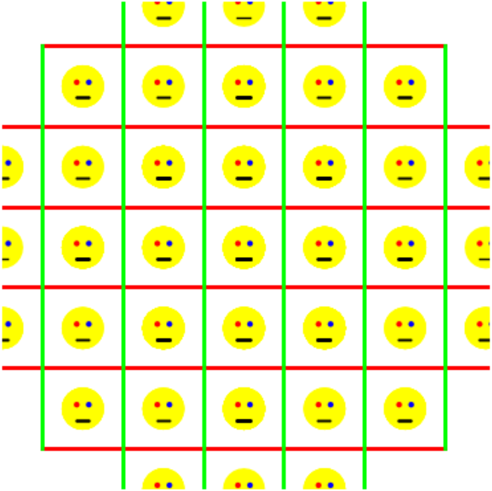

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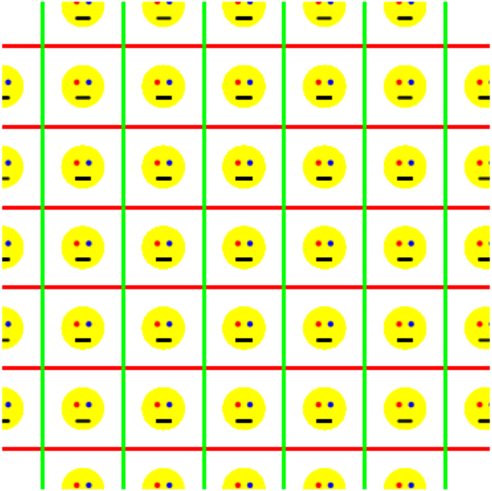
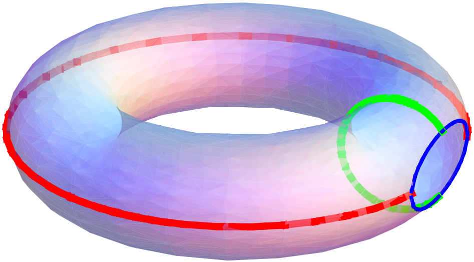


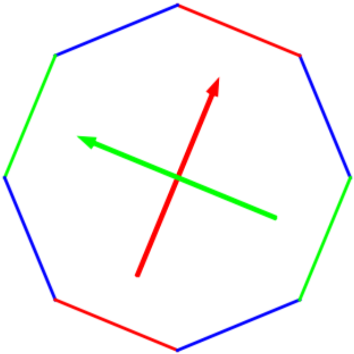
The Punctured Torus



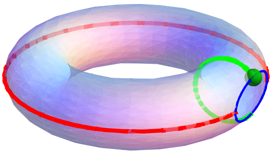
Remove an open disk from a torus. Assume the boundary is a geodesic.

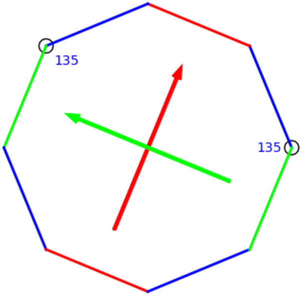
The Punctured Torus

Cut along those curves we obtain an octagon with piecewise geodesic boundary.



Problem!!!

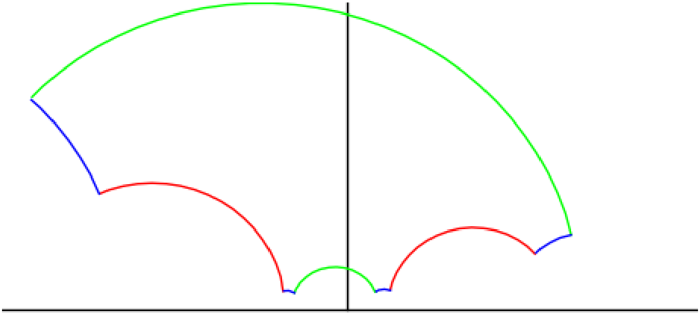




- Need 180° around a vertex
- Gluing 2 vertices of a Euclidean octagon yields 270° around that vertex.

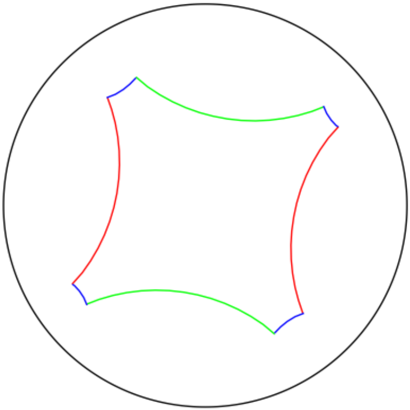
A Right angled Octagon

We can construct a right angled octagon in the Hyperbolic Plane

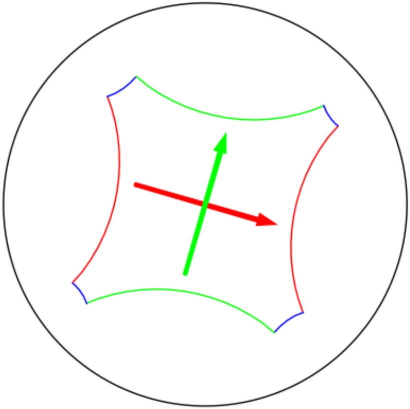


A Right angled Octagon

The same octagon in the Poincare Disk

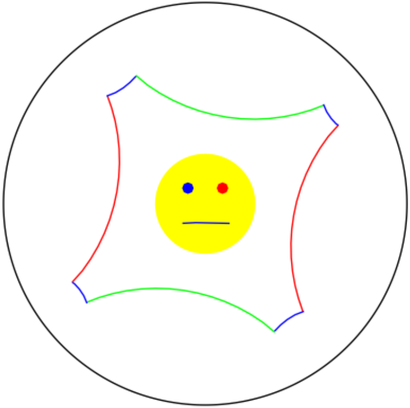


The punctured torus is obtained by gluing opposite sides.



These mappings are hyperbolic isometries

Build the Universal Cover for the Punctured Torus



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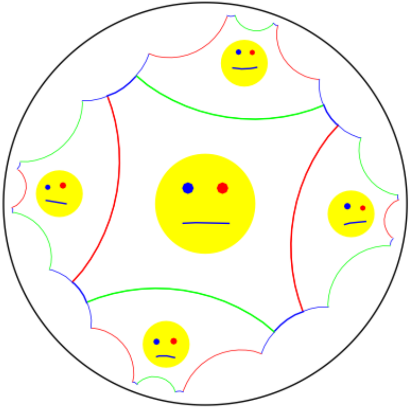

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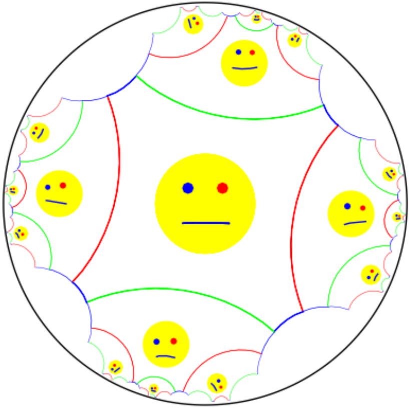

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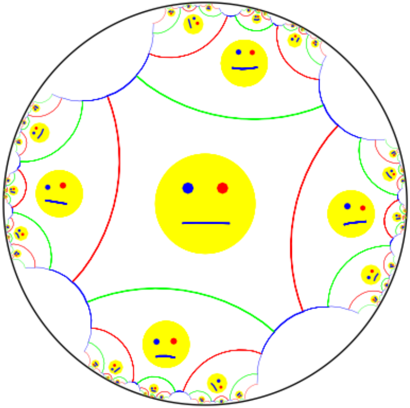

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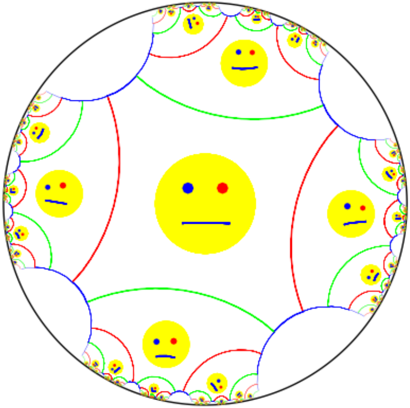
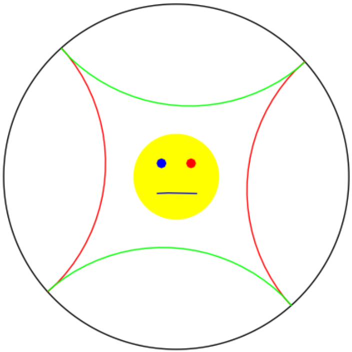
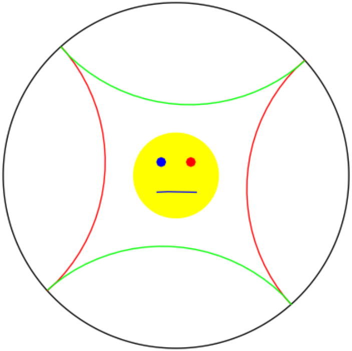


A Punctured Torus with a cusp

Remove a single point, we obtain a torus with a cusp. We obtain a structure by starting with an ideal quadralateral:



Universal Cover of a cusped torus



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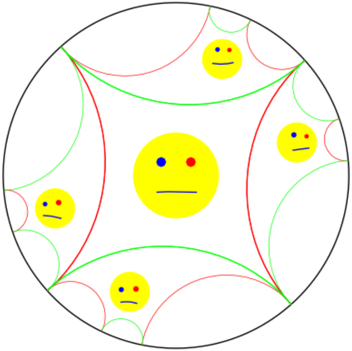

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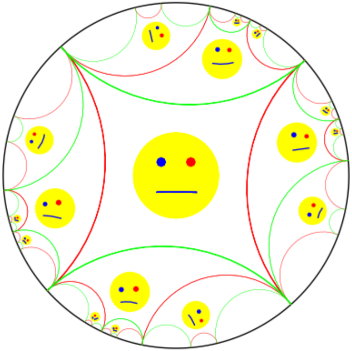

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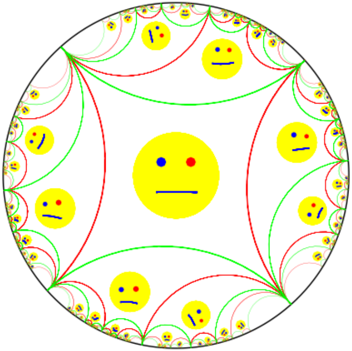

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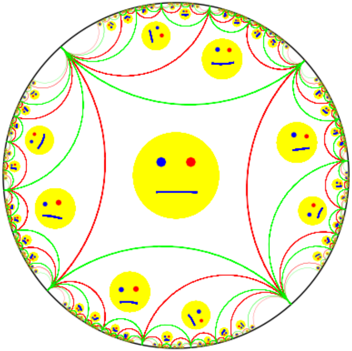

Shameless plug:

Check out the **Experimental Geometry Lab** at the University of Maryland: http://egl.math.umd.edu