Math 142

Homework 6 – Due March 13, 2018

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1. Do the following problems from Armstrong:
   - Page 85 #27, #29 (for #27, the action does not have to be "nice", so there can be x in the torus and $g \neq e$ such that $f_g(U) \cap U \neq \emptyset$ for all neighbourhoods U of x)
   - Page 102 #22 (see example 3 on page 81)
2. Let D^2 be the unit disc in $\mathbb{R}^2$, and let $S^1 = \partial D^2$ be its boundary (recall, ∂ means boundary). Show that there does not exist a continuous function $f : D^2 \to S^1$ such that $f(x) = x$ for all $x \in S^1 \subseteq D^2$.

   Hint: think about the inclusion map $i : S^1 \to D^2$.
3. Let $p : \widetilde{X} \to X$ be a covering space map. Recall that this means that for every $x \in X$, there exists an open neighbourhood U of x such that each component of $p^{-1}(U)$ is homeomorphic via p to U.
   (a) Let $x \in X$, and choose $\tilde{x} \in \widetilde{X}$ such that $p(\tilde{x}) = x$. Use the homotopy lifting lemma to show that $p_* : \pi_1(\widetilde{X}, \tilde{x}) \to \pi_1(X, x)$ is injective.
   (b) Using question 2a or otherwise, show that if T^2 is the torus and K is the Klein bottle, then there doesn't exist a covering space map $K \to T^2$.
4. Let $p : \mathbb{C} \to \mathbb{C}\backslash\{0\}$ be the map $p(z) = e^z$.
   (a) Show that p is a covering space map.
   (b) Find a nice action of $\mathbb{Z}$ on $\mathbb{C}$ such that $p(f_n(z)) = p(z)$ for all $n \in \mathbb{Z}$ and all $z \in \mathbb{C}$, and such that $p(z) = p(z')$ if and only if there is some $n \in \mathbb{Z}$ such that $f_n(z) = z'$.
   (c) Use (b) to conclude that $\pi_1(\mathbb{C}\backslash\{0\}) \cong \mathbb{Z}$. (Although we worked out $\pi_1(\mathbb{R}^2\backslash\{(0,0)\}$ in class, do not use this fact here.)
5. Prove that no two of the spaces S^2, S^1, or $S^0 = \{-1, 1\} \subseteq \mathbb{R}$ are homotopy equivalent. Hence, or otherwise, prove that no two of the spaces $\mathbb{R}^3$, $\mathbb{R}^2$, and $\mathbb{R}$ are homeomorphic.