Math 142

Homework 5 – Due March 6, 2018

Jamie Conway

1. Do the following problems from Armstrong:
   - Page 95 #11, #12
   - Page 102 #21
   - Page 109 #32
2. We can think of $\pi_1(X, p)$ as maps from S^1 (thought of as the unit circle in $\mathbb{C}$) to X that take $1 \in S^1$ to $p \in X$. Let $[S^1, X]$ be the set of homotopy classes of maps $S^1 \to X$ (that don't necessarily map 1 to p). There is a natural map

   $$\phi : \pi_1(X, p) \to [S^1, X]$$

   that just ignores the basepoint data.

   (a) Show that ϕ is surjective if X is path-connected.

   (b) Show that $\phi(\alpha) = \phi(\beta)$ if and only if there is some $g \in \pi_1(X, p)$ such that $\alpha = g\beta g^{-1}$.
3. Given spaces X and Y and two maps $f, g : X \to Y$, an *ambient isotopy* between f and g is a continuous function $F : Y \times [0, 1] \to Y$ such that $F_t : Y \to Y$ is a homeomorphism for every $t \in [0, 1]$ (where $F_t(y) = F(y, t)$), and $F_0 = \mathrm{id}_Y$ is the identity map, and $F_1 \circ f = g$.

   (a) If f and g are ambient isotopic, show that they are homotopic.

   (b) If f and g are homotopic, are they ambient isotopic? Why or why not?
4. Let X be a space, and let γ and σ be two paths that start at $p \in X$ and end at $q \in X$. Recall that these paths induce isomorphisms γ_* and σ_* from $\pi_1(X, p)$ to $\pi_1(X, q)$. Show that there exists an element $g \in \pi_1(X, q)$ such that

   $$\sigma_*(\alpha) = g\gamma_*(\alpha)g^{-1}$$

   for all $\alpha \in \pi_1(X, p)$. *Hint: write out the definitions of γ_* and σ_* and see what separates them. For a bigger hint, see Armstrong page 95 #10.*