Math 142

Homework 3 – Due February 13, 2018

Jamie Conway

1. Do the following problems from Armstrong:
   - Page 46 #1; page 47 #3
   - Page 50 #14
   - Page 60 #32
   - Page 73 #10 (use definition (a) of the projective plane ($n = 2$) from page 71)
2. Let X be a set, and let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two topologies on X, and suppose $\mathcal{T}_1 \subseteq \mathcal{T}_2$.
   (a) If X is compact (respectively, connected) in $\mathcal{T}_1$, is it compact (respectively, connected) in $\mathcal{T}_2$?
   (b) If X is compact (respectively, connected) in $\mathcal{T}_2$, is it compact (respectively, connected) in $\mathcal{T}_1$?
3. Show that in a space X with the discrete topology, the only connected components are singleton sets $\{x\}$, for $x \in X$. Show that for the set of rational numbers $\mathbb{Q} \subseteq \mathbb{R}$ with the subspace topology, the only connected components are singleton sets.

   This property is called being *totally disconnected*.
4. A *topological group* is a Hausdorff topological space G that is also a group, and where the multiplication function $G \times G \to G$ (sending (a, b) to ab) and the inverse function $G \to G$ (sending a to a^{-1}) are continuous. *(see Armstrong, page 73)*

   If $A, B \subseteq G$ are compact subsets of a topological group G (not necessarily subgroups), then show that the product

   $$AB = \left\{ ab \,\middle|\, a \in A, b \in B \right\}$$

   is also compact.