Math 142

Homework 2 – Due February 6, 2018

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1. Do the following problems from Armstrong:
   - Page 50 #6 and #17 (see #15 for the definition of *locally compact*)
   - Page 55 #25 and #26

2. Let $f : X \to Y$ be a function between two sets. If Y is a topological space, show that
$$\left\{ f^{-1}(A) \subseteq X \,\middle|\, A \subseteq Y \text{ is open} \right\}$$
defines a topology on X, and show that it's the smallest topology on X such that f is continuous.

3. We can define two topologies on the set $\mathbb{R}^2$: $\mathcal{T}_1$ is the usual topology, and $\mathcal{T}_2$ is the product topology (on $\mathbb{R} \times \mathbb{R}$, coming from the usual topology on each copy of $\mathbb{R}$). Show that $\mathcal{T}_1 = \mathcal{T}_2$.

   Hint: you are trying to prove that $U \in \mathcal{T}_1$ if and only if $U \in \mathcal{T}_2$, that is, $U \subseteq \mathbb{R}^2$ is open in the topology $\mathcal{T}_1$ if and only if it is open in the topology $\mathcal{T}_2$.

4. Let X be a topological space, and let A and B be two subspaces of X such that $X = A \cup B$. Let $Z = A \cap B$, and write $j : Z \to B$ for the inclusion map (that is, $j(z) = z$ for all $z \in Z$). Denote by Y the identification space $A \cup_j B$, *ie.* the result of of attaching A to B along $A \cap B$ via the map j.
   (a) Show that there is a natural bijection of sets $f : Y \to X$.
   (b) Show that the identification topology on Y can be described as follows: $U \subseteq Y$ is open if and only if $f(U) \cap A$ is open in A and $f(U) \cap B$ is open in B.
   (c) Show that f is continuous.
   (d) Show that f is a homeomorphism if A and B are both open sets in X.
   (e) If $X = \mathbb{R}$, $A = (-\infty, 0]$, and $B = (0, \infty)$, is f a homeomorphism?

5. In class, we identified one or two pairs of edges of a square to build a cylinder, a Möbius strip, a torus, a projective plane, and a Klein bottle. Identify all the topological spaces (up to homeomorphism) that you can build from a square in this manner, *ie.* by identifying pairs of edges.