MATH 104
Homework 7 – Due April 13, 2017
Jamie Conway

A select number of these questions will be graded (although the *starred* questions are optional, and will not be graded). Feel free (and encouraged!) to work with your classmates on this homework and come and talk about them in office hours, but you **must** write up your own solutions. Indicate on your homework the set of people with whom you worked, if that set is non-empty.

1. Ross §19, page 152: Exercise 8
2. Ross §28, pages 230-1: Exercises 3bc, 6, 8
3. Ross §29, pages 239-40: Exercises 1abcde, 4, 5, 14
4. Let f and g be continuous on $[a,b]$ and differentiable on (a,b). We will prove a generalisation of the mean value theorem: there exists a $c \in (a,b)$ such that

$$(f(b)-f(a)) \cdot g'(c) = (g(b)-g(a)) \cdot f'(c).$$

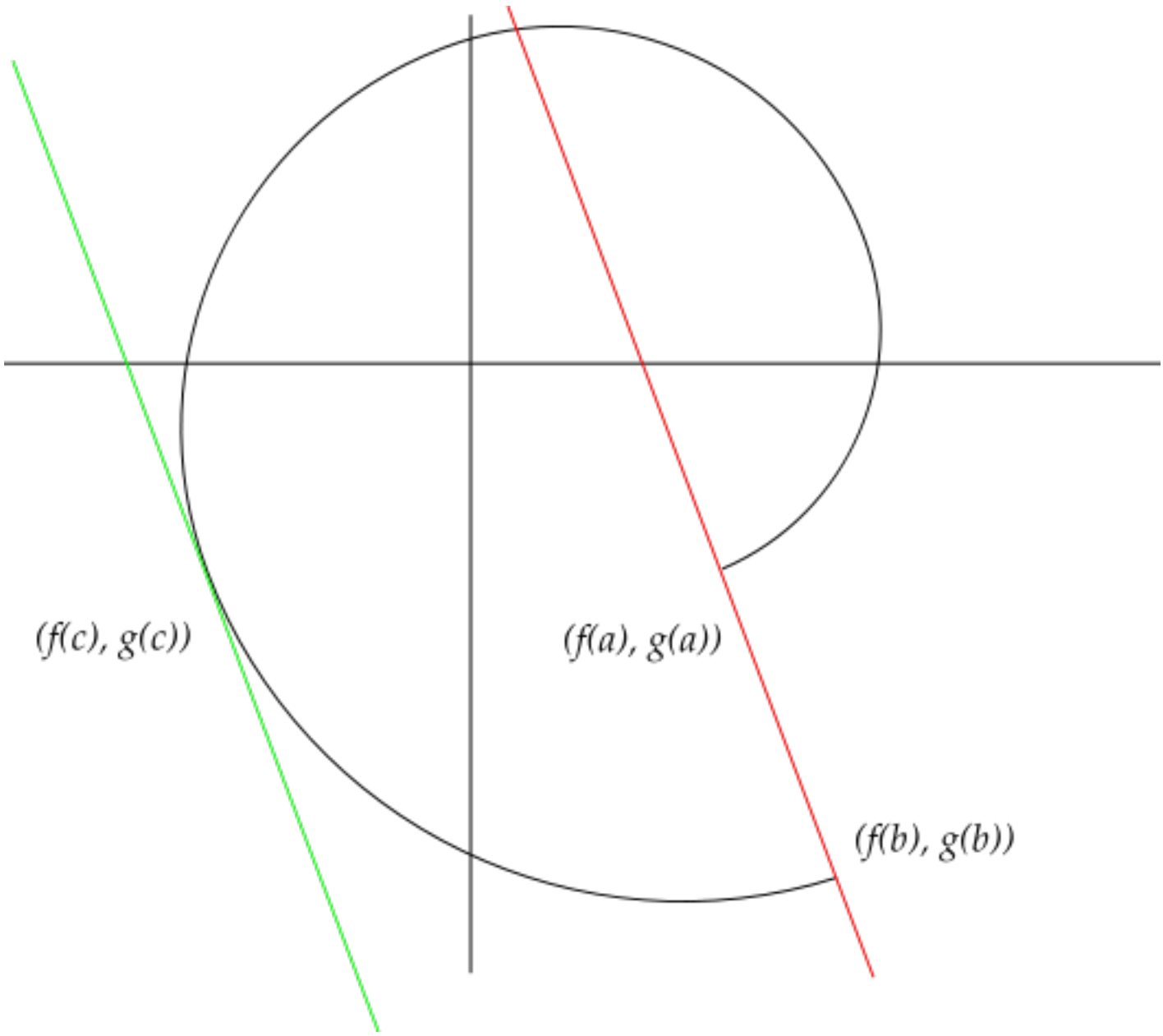
   See the picture on the next page for a geometric interpretation of this generalised mean value theorem.

   (a) If $g(a) = g(b)$, show that such a value of c exists.

   (b) If $g(a) \neq g(b)$, define a function $h(x) = f(x) - r \cdot g(x)$, for some constant $r \in \mathbb{R}$. Find a value of r such that $h(a) = h(b)$.

   (c) Since f and g are continuous on $[a,b]$ and differentiable on (a,b), so is h. Use Rolle's Theorem to show that there exists a c such that

$$(f(b)-f(a)) \cdot g'(c) = (g(b)-g(a)) \cdot f'(c).$$


(f(c), g(c))
(f(a), g(a))
(f(b), g(b))