

MATH 104
Homework 4 – Due February 28, 2017
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A select number of these questions will be graded (although the *starred* questions are optional, and will not be graded). Feel free (and encouraged!) to work with your classmates on this homework and come and talk about them in office hours, but you **must** write up your own solutions. Indicate on your homework the set of people with whom you worked, if that set is non-empty.

1. Ross §17, pages 130-132: Exercises 1, 9(abd), 10(ac), 13
2. In this question, we'll see that rearranging terms of alternating series can lead to interesting consequences. The two series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

and

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \dots$$

have the same terms in in different orders (the latter series has two positive terms for every negative term). Let s_n and t_n be the partial sums of these two series, respectively. Set

$$h_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}.$$

Show that

$$s_{2n} = h_{2n} - h_n$$

and

$$t_{3n} = h_{4n} - \frac{1}{2}h_{2n} - \frac{1}{2}h_n.$$

Show that (s_n) converges to a limit s , and t_n converges to $\frac{3s}{2}$.