

MATH 104  
Homework 3 – Due February 14, 2017  
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A select number of these questions will be graded (although the \*starred\* questions are optional, and will not be graded). Feel free (and encouraged!) to work with your classmates on this homework and come and talk about them in office hours, but you **must** write up your own solutions. Indicate on your homework the set of people with whom you worked, if that set is non-empty.

1. Ross §10, page 65: Exercises 6, 7
2. Ross §11, pages 76-77: Exercises 2, 7, 8, 10
3. Ross §12, page 82: Exercises 1, 6
4. Fix some real numbers  $a_1 > b_1 > 0$ , and then for every  $n \geq 1$ , define two sequences  $(a_n)$  and  $(b_n)$  by:

$$a_{n+1} = \frac{a_n + b_n}{2}, \quad b_{n+1} = \frac{2a_nb_n}{a_n + b_n}.$$

- (a) Show that  $a_n > a_{n+1} > b_{n+1} > b_n$ , for every  $n$ .
- (b) Deduce that  $(a_n)$  and  $(b_n)$  converge. If  $a_n \rightarrow a$  and  $b_n \rightarrow b$ , show that  $a = b$ .
- (c) What is that limit?