Math 130
Homework 6 – Due October 17, 2017
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1. Think about your independent project. Do research to help you choose a topic. (nothing to hand in)

2. Do the following problems from Stillwell: 5.6.1 – 5.6.4.

   Do the set of fractional linear transformations form a group? Explain.

3. Let $[p, q; r, s] = \frac{(p-r)(q-s)}{(q-r)(p-s)}$.

   (a) Show that $[p, q; r, s] \cdot [p, q; s, r] = 1$.

   (b) For which other permutations of the inputs is $[p, q; r, s] \cdot [?, ?; ?, ?] = 1$? Fill in the ?s with p, q, r, and s.

   (c) What is the relationship between $[p, q; r, s]$ and $[p, r; q, s]$?

   (d) For which other permutations does the relationship in (c) hold?

4. The *order* of a finite projective plane is one less than the number of points on a line. Show that this number is well-defined, *i.e.* that each line has the same number of points. *Hint: given two lines ℓ_1 and ℓ_2, find a bijection $\ell_1 \to \ell_2$.*

5. If we have a geometry G, then define the *dual geometry* $G!$ such that the set of points of $G!$ are the set of lines of G, and the lines of $G!$ are the points of G. A point of $G!$ is on a line of $G!$ if the corresponding line of G contains the corresponding point of G.

   (a) If G is a projective plane, show that $G!$ also satisfies the projective plane axioms. *Hint: use the result from HW5 that every point of G has at least 3 lines passing through it.*

   (b) A consequence of (a) is that every theorem in projective geometry has a *dual* theorem. That is, every statement about G can be converted into a statement about $G!$ by switching mentions of points and lines. What is the dual theorem to question 4 above?

   (c) (hard; not to hand in) If G is a projective plane, when is $G!$ isomorphic to G? Here, isomorphic means that there's a bijection f : points(G) $\to$ points(G!) such that the lines of G get mapped to the lines of $G!$.